Lecture 2
Time Value of Money

Chapter 5/122

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Topics Covered

- Understand Time Value of Money
- Simple Interest and Compound Interest
- Present Value and Future Value
- Multiple Cash Flows
- Perpetuities and Annuities
- Inflation & Time Value
- Effective Annual Interest Rate

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Time Value of Money

- This is the basic principle of finance that a dollar received today is more valuable than a dollar received in the future. Why would an investor choose to take the money today if he had the choice between today and next year?
- Money has time value because of interest rate, risk, expected inflation.

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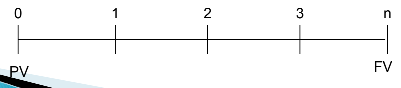
Terminology

- Simple Interest – Interest earned only on the original investment.



5.1.Future Value

Example – Simple Interest

Interest earned at a rate of 6% for five years on a principal balance of $100.

Interest Earned Per Year = 100 x 0.06 = $ 6

Future Value (Contd)

Example – Simple Interest (continued)

Interest earned at a rate of 6% for five years on a principal balance of $100.

	Today	Future Years				
		1	2	3	4	5
Interest Earned		**6**	**6**	**6**	**6**	**6**
Value	100	**106**	**112**	**118**	**124**	**130**

Value at the end of Year 5 = $130

Future Value (Contd)

Simple Interest

Assume that principal (P) is borrowed today at interest rate r, repayment after t periods. What would be the future sum?

$$S = P(1 + rt)$$

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Future Value (Contd)

Compound Interest – Interest earned on interest.

Example – Compound Interest

Interest earned at a rate of 6% for five years on the previous year's balance.

Interest Earned Per Year = Prior Year Balance x 0.06

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Future Value (Contd)

Example – Compound Interest

Interest earned at a rate of 6% for five years on the previous year's balance.

	Today	Future Years				
		1	2	3	4	5
Interest Earned		6	6.36	6.74	7.15	7.57
Value	100	106	112.36	119.10	126.25	133.82

Value at the end of Year 5 = $133.82

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Future Value (Contd)

- Future Value: Amount to which an investment will grow after earning interest.

Compound Interest

$$FV = P \times (1 + r)^t$$

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Giá trị tương lai

Lãi kép

$$FV = P \times (1 + r)^t$$

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Future Value (Contd)

Example – FV

What is the future value of $100 if interest is compounded annually at a rate of 6% for five years?

$$FV = \$100 \times (1 + 0.06)^5 = \$133.82$$

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Ex 1

- Computer the future value of a $100 cash flow with r= 8% ; t=20 years.

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Ex 2

- How long will it take for $400 to grow to $ 1,000 at the interest rate specified 8%

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Ex 3

- If you earn 6% per year on your bank account, how long will it take an account with $100 to double to $200?

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5.2.Present value

- To calculate present value, we discounted the future value at the interest rate. The calculation is thereforce termed a discounted cash-flow(DCF) calculation, and the interest rate is known as the discount rate.

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Present Value

Present Value
Value today of a future cash flow.

Discount Factor
Present value of a $1 future payment.

Discount Rate
Interest rate used to compute present values of future cash flows.

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Present Value (Contd)

Present Value = PV

$$PV = \frac{FV}{(1+r)^t}$$

Discount Factor = DF = PV of $1

$$DF = \frac{1}{(1+r)^t}$$

Discount Factors can be used to compute the present value of any cash flow.

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Present Value (Contd)

Example

You just bought a new computer for $3,000. The payment terms are 2 years. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?

$$PV = \frac{3000}{(1.08)^2} = \$2,572$$

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Applications of Time value of money

Present Value - The current value of one or more future cash flows from an investment.

$$\text{Present Value} = \frac{FV}{(1+r)^t}$$

Where FV is the future value, r is the rate of interest and t is the number of years.

The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable.

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Ex1:

- Would you rather receive $1,000 a year for 10 years or $800 a year for 15 years if the interest rate is 5%

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Ex2

- You will require $700 in 5 years. If you 5% interest on your funds, how much will you need to invest today in order to reach your savings goal?

Applications of Time value of money (Contd)

- Value of Free Credit
- Implied Interest Rates
- Internal Rate of Return (IRR)
- Time necessary to accumulate funds

Compounding more than once per year

- When interest is compounded more frequently than annually (e.g., m times per year): monthly, quarterly, semi -annually:

$$FV = PV\,[1 + r/m]^{n \times m}$$

FV: Future Value

PV: Present Value

r: Annual Interest rate (% p.a.)

n: number of years

m: times per year

Applications of Time value of money (Contd)

- Time necessary to accumulate funds:

Example

Suppose that you spend $10,000 to buy an car. If the market interest rate were 10%. How long does it take to receive payment of $50,000?

FV = 10,000 x $(1+0,1)^t$ = 50,000

t = ln5/ln(1,1) = 16,89 years

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Compounding more than once per year (Contd)

Example

You deposit $10,000 to a bank for 3 years. The interest rate is 6% p.a. How much money do you receive after 3 years if the interest is compounded:

a. semi-annually *b. quarterly* *c. monthly*

a. FV = $10,000 $[1+0,06/2]^6$ = $11,940.52
b. FV = $10,000 $[1+0,06/4]^{12}$ = $11,956.18
c. FV = $10,000 $[1+0,06/12]^{36}$ = $11,966.81

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Ex 1

- You invest $2,000 at a 8% annual interest rate, stated as an APR. Interest is compounded monthly. How much will you have in 1 year? In 1.5 years?

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Ex 2

- You invest $1,000 today and expect to sell your investment for $2,000 in 10 years.
- A. is this a good deal if the discount rate is 6%?
- B. What if the discount rate is 10%?

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5.3.Multiple Cash Flows

- We have considered problems involving only a single cash flow
- Investment will involve many cash flows overtime. When there are many payments, you'll hear manager refer to a stream of cashflows

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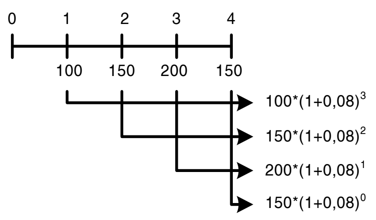
Ex :



Formula:
$FV = A1(1+r)^{t-1} + A2(1+r)^{t-2} + A3(1+r)^{t-3} + \ldots + An(1+r)^{0}$

FV :future value
A1, A2, … : payment in 1,2,3…
r: interest rate

Example

Company gives you the choice to pay $10,000 cash now, or make payments: $3,000 at the end of the following this year and $7,000 at the end of the following next year. If your interest rate 10%, which do you prefer?

PV of Multiple Cashflows

Example

Your auto dealer gives you the choice to pay $15,500 cash now, or make three payments: $8,000 now and $4,000 at the end of the following two years. If your cost of money is 8%, which do you prefer?

$$\text{Immediate payment } 8{,}000.00$$
$$PV_1 = \frac{4{,}000}{(1+.08)^1} = 3{,}703.70$$
$$PV_2 = \frac{4{,}000}{(1+.08)^2} = 3{,}429.36$$
$$\text{Total PV} = \$15{,}133.06$$

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PV of Multiple Cashflows (Contd)

- PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$

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Ex 1

- What is the present value of the following cash-flow stream if the interest rate is 6%

year	cash flow
1	$300
2	$500
3	$400

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PV of Multiple Cashflows (Contd)

Finding the present value of multiple cash flows by using a spreadsheet				
Time until CF	Cash flow	Present value	Formula in Column C	
0	8000	$8,000.00	=PV(B11,A4,0,-B4)	
1	4000	$3,703.70	=PV(B11,A5,0,-B5)	
2	4000	$3,429.36	=PV(B11,A6,0,-B6)	
SUM:		$15,133.06	=SUM(C4:C6)	
Discount rate:	0.08			

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Ex 3

Example

Your autocar gives you the choice to pay $20,000 cash now, or make three payments: $7,000 now and $8,000 at the end of the following two years. If your cost of money is 10%, which do you prefer?

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5.4.Perpetuities & Annuities

- **Perpetuity**

  A stream of level cash payments that never ends.

- **Annuity**

  Equally spaced level stream of cash flows for a limited period of time.

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Perpetuities & Annuities (Contd)

PV of Perpetuity Formula

$$PV = \frac{C}{r}$$

C = cash payment

r = interest rate

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Perpetuities & Annuities (Contd)

Example – Perpetuity

In order to create an endowment, which pays $100,000 per year, forever, how much money must be set aside today in the rate of interest is 10% ?

$$PV = \frac{100,000}{0.10} = \$1,000,000$$

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Ex 1

- A local bank advertises the following deal"will pay you$100 a year for your lifetime if you deposit $2,500 in the bank today.If you plan to live forever, what interest rate is the bank paying?

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Ex 2

- A property will provide $10,000 a year forever. If its value is $125,000, what must be the discount rate?

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Perpetuities & Annuities (Contd)

PV of Annuity Formula

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

With:
C = cash payment
r = interest rate
t = number of years cash payment is received

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Perpetuities & Annuities (Contd)

PV Annuity Factor (PVAF) - The present value of $1 a year for each of t years.

$$PVAF = \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

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Perpetuities & Annuities (Contd)

Example – Annuity

You are purchasing a car. You are scheduled to make 3 annual installments of $4,000 per year. Given a rate of interest of 10%, what is the price you are paying for the car (i.e. what is the PV)?

$$PV = 4{,}000\left[\frac{1}{.10} - \frac{1}{.10(1+.10)^3}\right]$$

$$PV = \$9{,}947.41$$

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Perpetuities & Annuities (Contd)

Applications:

- Value of payments
- Implied interest rate for an annuity
- Calculation of periodic payments
  - Mortgage payment
  - Annual income from an investment payout
  - Future Value of annual payments

$$FV = \left[C \times PVAF\right] \times (1+r)^t$$

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Perpetuities & Annuities (Contd)

Example – Future Value of annual payments

You plan to save $4,000 every year for 20 years and then retire. Given a 10% rate of interest, what will be the FV of your retirement account?

$$FV = 4{,}000\left[\frac{1}{.10} - \frac{1}{.10(1+.10)^{20}}\right] \times (1+.10)^{20}$$

$$FV = \$229{,}100$$

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Perpetuities & Annuities (Contd)

Annuity Due:

Level stream of cash flows starting immediately.

PV of an annuity due = (1+r) x PV of an annuity

FV of an annuity due = (1+r) x FV of an annuity

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Ex 1

- You've borrowed $4,248.68 and agreed to pay back the loan with monthly payments of $200. If the interest rate is 12% stated as an APR, how long will it take you to pay back the loan? What is the effective annual rate on the loan?

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5.5.Effective Interest Rate

Effective Annual Interest Rate(EAR)Interest rate that is annualized using compound interest.

$$1+ EAR = (1+\text{monthly rate})^{12}$$

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5.5.Effective Interest Rate

Annual Percentage Rate(APR):Interest rate that is annualized using simple interest.

Monthly interest rate = APR / 12

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Effective Interest Rate (Contd)

Example

Given a monthly rate of 1%, what is the Effective Annual Rate (EAR)? What is the Annual Percentage Rate (APR)?

$$EAR = (1+.01)^{12} - 1 = r$$

$$EAR = (1+.01)^{12} - 1 = .1268 \text{ or } 12.68\%$$

$$APR = .01 \times 12 = .12 \text{ or } 12.00\%$$

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Effective Interest Rate (Contd)

$$EAR = [1 + (r/m)]^{m \times n} - 1$$

Example

Consider the following interest rates quoted by three banks:

Bank A: 15%, compounded daily

Bank B: 15.5%, compounded quarterly

Bank C: 16%, compounded annually

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Effective Interest Rate (Contd)

Example:

Consider the following interest rates quoted by three banks:

Bank A: 15%, compounded daily

Bank B: 15.5%, compounded quarterly

Bank C: 16%, compounded annually

- EAR (A) = $[1 + 0.15/365]^{365} - 1$ = 16,18%
- EAR (B) = $[1 + 0.155/4]^{4} - 1$ = 16,42%
- EAR (C) = $[1 + 0.16/1]^{1} - 1$ = 16%

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Ex 1

Example:

Consider the following interest rates quoted by three banks:

Bank A: 15.5%, compounded daily

Bank B: 16%, compounded monthly

Bank C: 16%, compounded annually

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Ex 2

- If a banks pays 6% interest with continuos compounding, what is the effective annual rate?

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5.6.Inflation

- Inflation – Rate at which prices as a whole are increasing.
- Nominal Interest Rate – Rate at which money invested grows.
- Real Interest Rate – Rate at which the purchasing power of an investment increases.

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Inflation (Contd)

$$1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

Approximation formula:

$$\text{Real int. rate} \approx \text{nominal int. rate} - \text{inflation rate}$$

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Inflation (Contd)

Example

If the interest rate on one year govt. bonds is 5.0% and the inflation rate is 2.2%, what is the real interest rate?

$1 + \text{real interest rate} = \frac{1+.050}{1+.022}$

$1 + \text{real interest rate} = 1.027$

$\text{real interest rate} = .027 \text{ or } 2.7\%$

$\text{Approximation} = .050 - .022 = .028 \text{ or } 2.8\%$

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Thanks for your attention

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